

MOVEMENT OF THE PARTICLE ON THE INTERNAL SURFACE OF THE SPHERICAL SEGMENT ROTATING ABOUT A VERTICAL AXIS

РУХ ЧАСТИНКИ ПО ВНУТРІШНІЙ ПОВЕРХНІ СФЕРИЧНОГО СЕГМЕНТА, ЯКИЙ ОБЕРТАЄТЬСЯ НАВКОЛО ВЕРТИКАЛЬНОЇ ОСІ

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DOI: <https://doi.org/10.35633/inmateh-62-08>

Keywords: angular velocity, relative motion, particle movement, differential equations, kinematic parameters.

ABSTRACT

The particle relative motion on a spherical segment rotating about a vertical axis was considered in the article. The differential equations of the relative displacement of a particle were completed and solved by numerical methods. The relative and absolute trajectories of particle motion and graphs of relative and absolute velocity changes were constructed. The regularity of particle motion as it is lifted over the surface was found out. The conducted experimental research has confirmed the received theoretical results.

АНОТАЦІЯ

Розглянуто відносний рух частинки по сферичному сегменту, який обертається навколо вертикальної осі. Складено диференціальні рівняння відносного переміщення частинки, які розв'язано чисельними методами. Побудовано відносну та абсолютну траєкторії руху частинки та графіки зміни відносної і абсолютної швидкостей. З'ясовано закономірність руху частинки при її підйомі по поверхні. Проведені експериментальні дослідження підтвердили отримані теоретичні результати.

INTRODUCTION

The theory of particle motion on surfaces rotating about a vertical axis is used for designing centrifugal action devices (Lytvynenko *et al.*, 2019). In particular, it concerns devices for the dispersal of mineral fertilizers (Khan *et al.*, 2018), the extraction of juice from vegetables and fruits (Mushtaq *et al.*, 2017), the purification of air from dust particles in cyclones (Galins *et al.*, 2018) or in the process of designing the vortex type liquid-vapour jet apparatus (Merzliakov *et al.*, 2020).

It is common knowledge, that the particle performs a complex motion, which is the sum of two motions: the transportable movement of the surface and the relative movement of the particle on the surface, that is, it's sliding. A great deal is being written and said about the motion of particles on the surface of the cylinder (Pylypaka *et al.*, 2018; Pylypaka *et al.*, 2019) and on the surface of the cone (Carpena *et al.*, 2014). Importantly, the movement of particles on other rotating surfaces has its own peculiarities (Ivanov *et al.*, 2019; Liu *et al.*, 2019). Based on the foregoing, the aim of our research is the investigation of the regularities of motion of a material particle on a spherical segment, which rotates about a vertical axis with constant angular velocity.

MATERIALS AND METHODS

Generally, the parametric equations of a sphere with the origin in its lower pole are written:

$$X = R \sin \varepsilon \cos \alpha$$

$$Y = R \sin \varepsilon \sin \alpha \tag{1}$$

$$Z = R(1 - \cos \varepsilon)$$

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where:

R is the radius of the sphere;

ε, α are the independent variables of the sphere (angles that define the position of a point on the sphere surface in the direction of the meridian and the parallel respectively).

Clearly, the angle α varies within limits $\alpha = 0 \dots 2\pi$, the angle ε – within $\varepsilon = 0 \dots \varepsilon_0$, where the height of the spherical segment depends on the value of the angle ε_0 . For example, at $\varepsilon_0 = \pi/2$ the segment will be equal to half of the sphere.

Segment rotation will be considered regarding two coordinate systems: the fixed $OXYZ$ and the moving $Oxyz$. The last one rotates with the segment. If the spherical segment (1) rotates about a vertical axis with an angular velocity ω , then in time t the surface will turn to the angle $\theta = \omega \cdot t$. Let apply the known formulas of rotation of one coordinate system relative to another:

$$\begin{aligned} X &= R \sin \varepsilon \cos \alpha \cos \theta - R \sin \varepsilon \sin \alpha \sin \theta \\ Y &= R \sin \varepsilon \cos \alpha \sin \theta + R \sin \varepsilon \sin \alpha \cos \theta \\ Z &= R(1 - \cos \varepsilon) \end{aligned} \tag{2}$$

Taking into account $\theta = \omega \cdot t$ after simplifications equations (2) can be written:

$$\begin{aligned} X &= R \sin \varepsilon \cos(\alpha + \omega t); \\ Y &= R \sin \varepsilon \sin(\alpha + \omega t); \\ Z &= R(1 - \cos \varepsilon). \end{aligned} \tag{3}$$

Firstly, at the initial moment ($t = 0$) the two coordinate systems coincide, the spherical segment (or disk) does not rotate and the particle is at the meridian in the OYZ plane (Figure 1,a). For this particular case, the following forces are applied to the particle: the force of gravity mg (m is the mass of the particle, $g=9.81 \text{ m/s}^2$ is the acceleration of gravity), surface reaction N , and the force of friction $f \cdot N$ (f is the coefficient of friction), which prevents the particle movement down along the meridian toward the origin. Indeed, as the disc rotates at a constant angular velocity ω during time t , it will rotate at an angle $\theta = \omega \cdot t$ (Figure 1,b). If the particle would not slide on the disk, it would rotate with the disk to the corner θ and would take a position on the same meridian after its rotation. As a result of sliding, the particle occupies an intermediate position (Figure 2,a). A particle sliding occurs in the opposite direction of rotation of the disk. So, the direction of relative velocity V_r is directed tangentially to the particle sliding trajectory (Figure 1,b).

The equation of particle motion should be derived from form $m\vec{w} = \vec{F}$, where \vec{w} is the vector of acceleration, \vec{F} is the resultant vector of applied forces to the particles. Hence, all vectors should be defined in the projections on the axis of the fixed coordinate system. The trajectory of the relative motion of a particle regarding the moving $Oxyz$ coordinate system will be described by the dependency between curvilinear coordinates of the sphere ε and α . Actually, this dependency can be set differently: $\varepsilon = \varepsilon(\alpha)$, $\alpha = \alpha(\varepsilon)$ or by means of a common variable t : $\varepsilon = \varepsilon(t)$, $\alpha = \alpha(t)$. In our particular case, the common variable is time t .

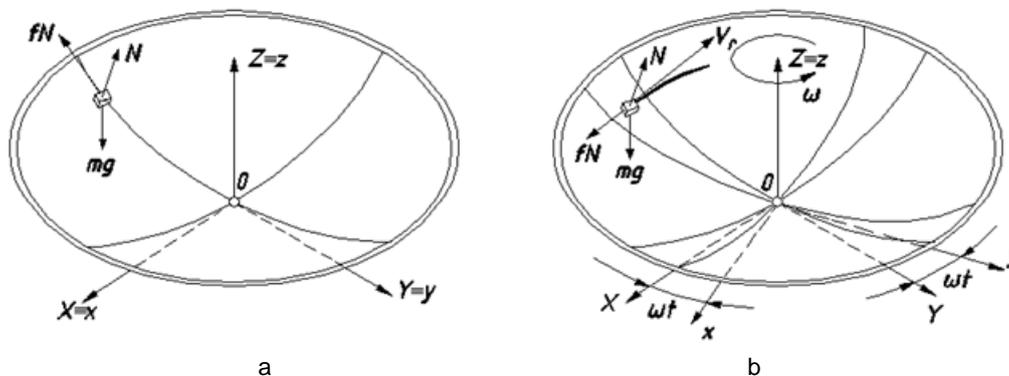


Fig. 1 - The location of the particle in the spherical segment and the scheme of forces applied to it
 a) the fixed and moving coordinate systems coincide, the segment does not rotate;
 b) the segment rotates, the particle slides on its surface

Thus, according to $\varepsilon = \varepsilon(t)$ and $\alpha = \alpha(t)$, equation (1) defines the relative trajectory of particle motion and equation (3) – the absolute trajectory.

These dependencies are unknown and should be found. Besides, the vectors of relative and absolute velocities should be determined by differentiating expressions (1) and (3) by time t . Equations (1) and (3) in one case are the surface equations when ε and α are the independent variables, and in the other, they are lines (particle trajectories which should be found) on the surface. For the surfaces the indication of the equations in uppercase letters was taken, as well as for the lines – the capital letters. For the relative trajectory the index "r" is used and for the absolute – "a". Therefore, by differentiating equations (1) the relative velocity of particle motion (sliding) on the surface of a spherical disk can be found:

$$\begin{aligned}\dot{x}_r &= R\dot{\varepsilon} \cos \varepsilon \cos \alpha - R\dot{\alpha} \sin \varepsilon \sin \alpha \\ \dot{y}_r &= R\dot{\varepsilon} \cos \varepsilon \sin \alpha + R\dot{\alpha} \sin \varepsilon \cos \alpha \\ \dot{z}_r &= R\dot{\varepsilon} \sin \varepsilon\end{aligned}\quad (4)$$

The geometric sum of the components (4) gives an opportunity to obtain the value of the sliding velocity of the particle on the spherical disk in the relative motion:

$$V_r = \sqrt{\dot{x}_r^2 + \dot{y}_r^2 + \dot{z}_r^2} = R\sqrt{\dot{\varepsilon}^2 + \dot{\alpha}^2 \sin^2 \varepsilon} \quad (5)$$

The unit vector T of the tangent to the trajectory of relative motion in projections on the axis of the OXYZ system is obtained by dividing the projections (4) to the value of the vector (5):

$$\begin{aligned}T_x &= \frac{\dot{\varepsilon} \cos \varepsilon \cos \alpha - \dot{\alpha} \sin \varepsilon \sin \alpha}{\sqrt{\dot{\varepsilon}^2 + \dot{\alpha}^2 \sin^2 \varepsilon}} \\ T_y &= \frac{\dot{\varepsilon} \cos \varepsilon \sin \alpha + \dot{\alpha} \sin \varepsilon \cos \alpha}{\sqrt{\dot{\varepsilon}^2 + \dot{\alpha}^2 \sin^2 \varepsilon}} \\ T_z &= \frac{\dot{\varepsilon} \sin \varepsilon}{\sqrt{\dot{\varepsilon}^2 + \dot{\alpha}^2 \sin^2 \varepsilon}}\end{aligned}\quad (6)$$

Let us find the direction of action of the surface reaction N (1). It is directed toward the normal P to the surface and is determined from the vector product of two vectors tangent to the coordinate lines of the surface. The projections of these vectors are partial derivatives of equations (1):

$$\begin{aligned}\frac{\partial X}{\partial \varepsilon} &= R \cos \varepsilon \cos \alpha; & \frac{\partial Y}{\partial \varepsilon} &= R \cos \varepsilon \sin \alpha; & \frac{\partial Z}{\partial \varepsilon} &= R \sin \varepsilon; \\ \frac{\partial X}{\partial \alpha} &= -R \sin \varepsilon \sin \alpha; & \frac{\partial Y}{\partial \alpha} &= R \sin \varepsilon \cos \alpha; & \frac{\partial Z}{\partial \alpha} &= 0.\end{aligned}\quad (7)$$

After vector multiplication of vectors (7) and transformation of the obtained vector into a unit one, projections of the vector of the normal P to the surface are:

$$P_x = -\sin \varepsilon \cos \alpha; \quad P_y = -\sin \varepsilon \sin \alpha; \quad P_z = \cos \varepsilon. \quad (8)$$

By differentiation of equations (3), the absolute velocity of motion of a particle with respect to a fixed coordinate system can be written:

$$\begin{aligned}\dot{x}_a &= R\dot{\varepsilon} \cos \varepsilon \cos(\alpha + \omega t) - R(\dot{\alpha} + \omega) \sin \varepsilon \sin(\alpha + \omega t) \\ \dot{y}_a &= R\dot{\varepsilon} \cos \varepsilon \sin(\alpha + \omega t) + R(\dot{\alpha} + \omega) \sin \varepsilon \cos(\alpha + \omega t) \\ \dot{z}_a &= R\dot{\varepsilon} \sin \varepsilon\end{aligned}\quad (9)$$

Then, the projections of the vector of absolute acceleration on the axis of the fixed coordinate system can be obtained by differentiation of expressions (9):

$$\begin{aligned}
\ddot{x}_a &= R \left[\ddot{\varepsilon} \cos \varepsilon - \dot{\varepsilon}^2 \sin \varepsilon - (\dot{\alpha} + \omega)^2 \sin \varepsilon \right] \cos(\alpha + \omega t) - \\
&\quad - R \left[\ddot{\alpha} \sin \varepsilon + 2\dot{\varepsilon}(\dot{\alpha} + \omega) \cos \varepsilon \right] \sin(\alpha + \omega t) \\
\ddot{y}_a &= R \left[\ddot{\varepsilon} \cos \varepsilon - \dot{\varepsilon}^2 \sin \varepsilon - (\dot{\alpha} + \omega)^2 \sin \varepsilon \right] \sin(\alpha + \omega t) + \\
&\quad + R \left[\ddot{\alpha} \sin \varepsilon + 2\dot{\varepsilon}(\dot{\alpha} + \omega) \cos \varepsilon \right] \cos(\alpha + \omega t) \\
\ddot{z}_a &= R\ddot{\varepsilon} \sin \varepsilon + R\dot{\varepsilon}^2 \cos \varepsilon
\end{aligned} \tag{10}$$

The unit vector T (6) of the relative velocity direction V_r and the unit vector (8) of the normal to the surface P are found for the fixed surface. Since the surface rotates at an angle $\theta = \omega \cdot t$, the vectors also should be rotated at this angle to correspond to the particle location. The rotation is carried out in the same way as the rotation of the surface according to formulas (2). After rotation, the projections of these vectors can be written:

- a unit vector of the tangent to the relative trajectory:

$$\begin{aligned}
T_X &= \frac{\dot{\varepsilon} \cos \varepsilon \cos(\alpha + \omega t) - \dot{\alpha} \sin \varepsilon \sin(\alpha + \omega t)}{\sqrt{\dot{\varepsilon}^2 + \dot{\alpha}^2 \sin^2 \varepsilon}} \\
T_Y &= \frac{\dot{\varepsilon} \cos \varepsilon \sin(\alpha + \omega t) + \dot{\alpha} \sin \varepsilon \cos(\alpha + \omega t)}{\sqrt{\dot{\varepsilon}^2 + \dot{\alpha}^2 \sin^2 \varepsilon}} \\
T_Z &= \frac{\dot{\varepsilon} \sin \varepsilon}{\sqrt{\dot{\varepsilon}^2 + \dot{\alpha}^2 \sin^2 \varepsilon}}
\end{aligned} \tag{11}$$

- a unit vector of the normal to the surface:

$$P_X = -\sin \varepsilon \cos(\alpha + \omega t); \quad P_Y = -\sin \varepsilon \sin(\alpha + \omega t); \quad P_Z = \cos \varepsilon. \tag{12}$$

At the same time, the vector equation $m\overline{w} = \overline{F}$ in the projections on the axis of the fixed coordinate system OXYZ is written:

$$\begin{aligned}
m\ddot{x}_a &= NP_x - fNT_x \\
m\ddot{y}_a &= NP_y - fNT_y \\
m\ddot{z}_a &= NP_z - fNT_z - mg
\end{aligned} \tag{13}$$

The projections of unit directional vectors of tangent T to the relative trajectory and normal to the surface P are given in (11) and (12) accordingly, and the expressions of the second derivatives of the absolute trajectory are given in (10). As a result, substituting the expressions in (13) gives a system of three equations with three unknown dependencies: $\alpha = \alpha(t)$, $\varepsilon = \varepsilon(t)$, and $N = N(t)$. It is essential to solve it regarding $\ddot{\alpha}$, $\ddot{\varepsilon}$ and N :

$$\begin{aligned}
\ddot{\varepsilon} &= \left[(\omega + \dot{\alpha})^2 \cos \varepsilon - \frac{g}{R} \right] \sin \varepsilon - f \frac{\dot{\varepsilon} B}{RA} \\
\ddot{\alpha} &= -2\dot{\varepsilon}(\omega + \dot{\alpha}) \operatorname{ctg} \varepsilon - f \frac{\dot{\alpha} B}{RA} \\
N &= mB
\end{aligned} \tag{14}$$

where: $A = \sqrt{\dot{\varepsilon}^2 + \dot{\alpha}^2 \sin^2 \varepsilon}$; $B = g \cos \varepsilon + R \left[\dot{\varepsilon}^2 + (\omega + \dot{\alpha})^2 \sin^2 \varepsilon \right]$.

Actually, system (14) is the system of the first two equations relatively unknown dependencies $\alpha = \alpha(t)$ and $\varepsilon = \varepsilon(t)$, and the dependence $N = N(t)$ can be found after the solution of this system. The resulting system can be solved by numerical methods. It is necessary to substitute the found dependencies $\alpha = \alpha(t)$ and $\varepsilon = \varepsilon(t)$ into equation (1) in order to obtain the relative trajectory of particle motion on a spherical disk, that is, the sliding trajectory, and into equation (3) to obtain the absolute trajectory movement.

RESULTS

First of all, in Figure 2 by numerical methods the relative and absolute trajectories of particle motion on the disk for 2 s period were constructed with the following parameters: $R=0.5\text{ m}$, $\omega=20\text{ s}^{-1}$, $f=0.3$.

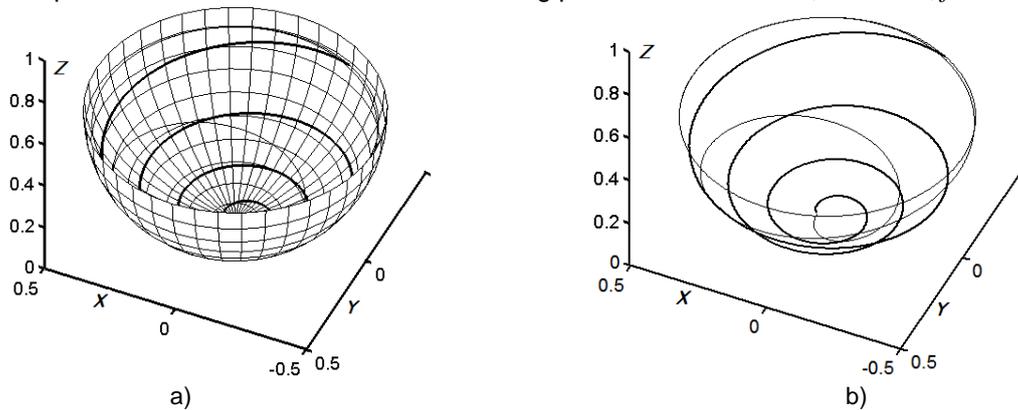


Fig. 2 - The relative (thick line) and the absolute trajectory of particle motion
 a) on the surface of the spherical segment; b) the surface of the segment conventionally is not shown

After analysing the trajectory, it can be concluded that the particle slides on the sphere, rising to a certain position, then "sticks" and rotates with the surface. This case is shown on the graphs of the relative and absolute velocities of the particles (Figure 3). The absolute velocity of a particle V_a is defined as the geometric sum of its projections (9) by the formula (5).

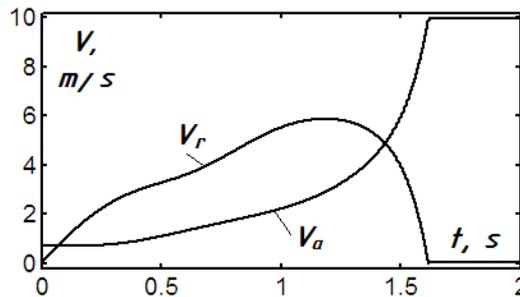


Fig. 3 - Graphs of relative V_r and absolute V_a velocities of particle motion

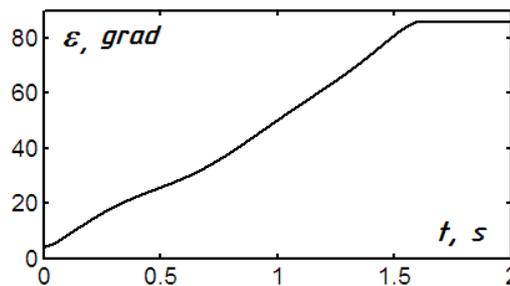


Fig. 4 - Graph of the angle change ϵ

Moreover, Figure 3 shows that after 1.6 s period after the start of the motion, the particle "sticks", so, its sliding speed becomes zero and the absolute speed becomes equal $V_a = 10\text{ m/s}$. From the graph of change of angle ϵ (Figure 4) it is seen that at the moment of particle sticking at $t = 1.6\text{ s}$, it reaches the maximum value $\epsilon = 85^\circ$, so, the particle with a segment rotates by a circle that is almost equal to the equator of the sphere, that is, the radius of the sphere R . Based on the set values $R = 0.5\text{ m}$ and $\omega=20\text{ s}^{-1}$, one can find: $V_a = \omega \cdot R = 10\text{ m/s}$, that is, the obtained velocity is consistent with the graph. If a particle begins the movement not from the lower point of the segment, but little higher (for example, at $\epsilon_0 = 45^\circ$, as shown in Figure 5), it reaches soon the upper boundary trajectory, beyond the hemisphere. It requires 0.2 s (Figure 6). Figure 7 shows a graph of the change of the surface reaction for a particle by mass $m=0.01\text{ kg}$.

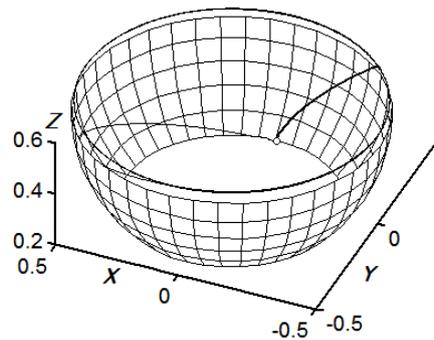


Fig. 5 - Relative (thick line) and absolute trajectory of motion on a bounded area of a segment

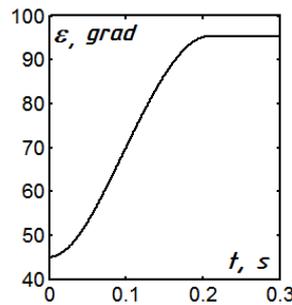


Fig. 6 - Graph of change of angle ε for initial value $\varepsilon_0=45^\circ$

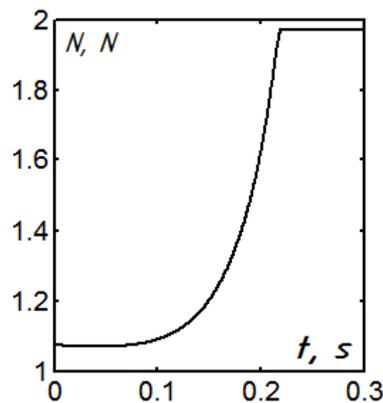


Fig. 7 - Graph of change of surface reaction for a particle with mass $m=0.01 \text{ kg}$

For instance, the movement of the particle on the inner wall of the rotating surface takes place in devices for juice squeezing from fruits or vegetables. Usually, a cut cone with holes for the juice outlet is used there. Doubtless, the movement of the particle of technological material on the inner surface of the cone is different from a similar movement on the surface of the segment. Sliding on the surface of the cone, the particle will rise up and the speed will increase.

The segment of the sphere is characterized by the deceleration of the velocity V_r of the particle sliding as it rises up to the "sticking" in the vicinity of the equator (Figure 3). But the most intensive juicing process occurs in the braking zone, as evidenced by the sharp increase in pressure on the particle (Fig.7).

Of course, the nature of the movement of the technological material, such as the core, is different from that particle movement. The considered model does not take into account the obstacles to movement, which are holes in the wall surface. Obviously, as the cone rotates, some of the material moves up and out of bounds without giving all the juice. The other part "sticks" and is separated from the surface by a mechanical device with a manual drive. However, the behaviour of the individual particle gives a qualitative characterization of the movement, which can be transferred to the movement of the material particularly. In particular, the applying of a spherical segment makes it possible to prevent unauthorized abandonment of particles from the work area material.

In order to confirm the theoretical research, we have made a device for squeezing juice from grated fruits, vegetables and fruits (squash), in which a segment of a sphere with holes for removing juice was used as a working surface.

The research program consisted of determining the mass fraction of squash moisture after juice removal or the quality of juice release from the squash. The studies were performed according to GOST 13979.1-88, which establishes methods for determining the mass fraction of squash moisture in the range of values from 2 to 20%, and consisted of drying samples of boxes with the product and determining their weight before and after drying.

The mass fraction of moisture was determined by the formula:

$$W = \frac{m_1 - m_2}{m_1 - m} \times 100\%, \quad (15)$$

where:

m is a mass of empty box;

m_1 is a mass of box with the product before drying;

m_2 is a mass of box with the product after drying.

The research was carried out by squeezing the juice from carrots, beets and apples squash both on a standard centrifugal juicer PHILIPS Viva Kompakt HR1832/02 and on a specially made device for squeezing juice with a segment of the sphere as a working surface.

It was found that the mass fraction of moisture in the squash squeezed in our experimental device is 8.1... 19.7 % less than in the squash squeezed by the PHILIPS juicer.

The results of experimental studies to determine the mass fraction of moisture in the squash of carrots, beets and apples are shown in Figure 8.

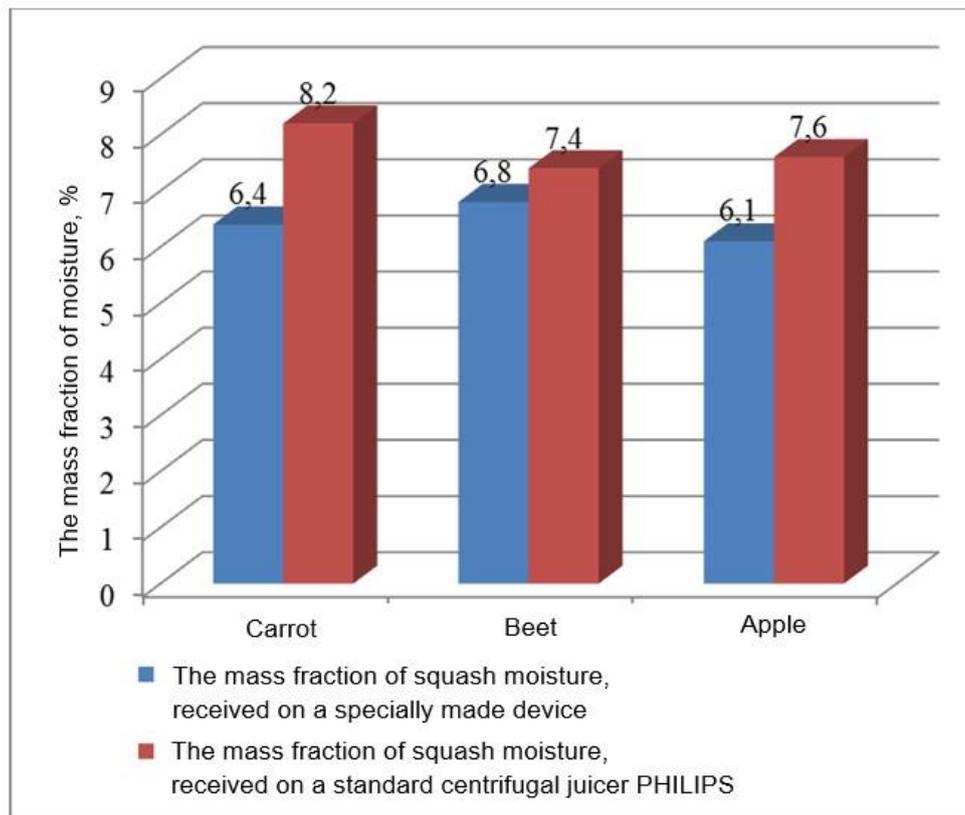


Fig. 8 - The results of experimental researches to determine the mass fraction of squash moisture

CONCLUSIONS

The particle motion on the inner rough surface of a spherical segment has its own peculiarities. It relates to the nature of the particle sliding across the surface. When a particle hits the surface at the bottom of the segment, its acceleration occurs with a simultaneous upward movement. Such motion is characterized by the change of two speeds: relative (the speed of sliding) and absolute. Firstly, the relative velocity increases and then decreases to zero at the time of a particle sticking. The

absolute velocity of a particle constantly increases and becomes constant after its "sticking". The "sticking" of the particle occurs at a height close to the equator of the sphere and it can stick both below and above depending on the initial conditions. The conducted experimental research has confirmed the received theoretical results.

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